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Magalie Roman Salas
Secretary
Federal Communications Commission
The Portals
445 12th St., SW
Washington, D.C.. 20554
USA

Re: Cc Docket No 94-102 - FCC E9-1-1 Order

Ex Parte Submission

Dear Ms. Salas,

Cambridge Positioning Systems Ltd. (CPS) has developed an ALI technology for mass market international deployment. Extensive technical trials to assess the performance and reliability of the ALI technology were completed on a GSM 900 MHz network in 1998. CPS is currently engaged in commercial trials with the UK's largest wireless operator, Vodafone, and the UK's largest vehicle breakdown services provider, the Automobile Association, which is also a major information service provider, to assess consumer demand and validate commercial applications for location-based services.

The ALI method developed by CPS is known as an Enhanced Observed Time Difference (E-OTD) system, which is a low complexity handset-based solution that requires only the addition of a small amount of software to the handset.

The US-based T1 group, specifically T1P1.5 LCS SWG (Location Services Sub Working Group) [*1*], is currently in the advanced stages of developing a standard for location systems to be implemented on a GSM network, in response to FCC Docket No. 94-102. Three different approaches are to be included in the standard, of which the two handset-based solutions are GPS and E-OTD. As such, the meaning of handset-based solutions should include the E-OTD technique as well as GPS. CPS' comments herein are therefore made with respect to both ALI technologies.

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During its initial formal pleading cycle the FCC sought comments on the accuracy standards that should apply to handset-based solutions. Objectives tests conducted on a deployed E-OTD system covering a 150 km^2 area included a mix of urban, suburban and rural environments and it demonstrated location accuracies consistently better than 125 m. CPS therefore supports a handset-based solution for the ALI technology required to support FCC Docket No. 94-102.

In addition to the numerous field trials performed by CPS, the company has investigated the means by which both the confidence region associated with a single location, and the accuracy of the ALI system, should be measured and has proposed a methodology to the T1P1.5 LCS SWG. The source data upon which this contribution is based is attached below.

The CEP (Circular Error Probable) method for measuring accuracy promoted by, e.g. SnapTrack [*3*], is not generalised, as it is likely to overestimate the uncertainty of a position. It is well known [*4*] that the confidence region associated with a location is better represented by an ellipse than a circle. Although a circle is a close enough approximation for many position fixes, the actual confidence region for both GPS and terrestrial positioning methods has a much more diverse variation of shape and a circle is not always appropriate. It is therefore proposed that a confidence ellipse, characterised by the size of its major and minor axes and the orientation of its major axis, be adopted as the standard estimate for the error in a position. Circular confidence regions may obviously still be specified, as the circle is a special case of an ellipse.

The methodology for determining the accuracy of the ALI system is as follows.

It is required that ALI technologies meeting the requirements of the applicable Commission's Rules must provide to the designated PSAP "the location of all 911 calls by longitude and latitude such that the accuracy for all calls is 125 metres or less using a Root Mean Square (RMS) methodology". Extensive trials conducted by CPS [*2, 5*] indicate that RMS is a biased estimator for the 67 percent confidence radius (within which 67% of all measurements lie). It has been suggested through filings and presentations by Ericsson and the Wireless E-911 Implementation Ad Hoc (WEIAD) group [*3*] that the RMS methodology should not apply to all E-911 calls. It is claimed by these companies that a small number of measurements that are very inaccurate will prevent a carrier from complying with the ALI requirements even if the vast majority of ALI measurements are less than 125 metres.

CPS has proven in its extended trials of an E-OTD ALI technology [*2*, *5*] that no measurement need be rejected in computing accuracy. It is demonstrated that the technique of ignoring a certain percentage of measurements gives very poor predictions when ten percent is chosen (as proposed for RMS_{90}). In fact, no single percentage of rejection can be adopted to generalise every possible set of data. The optimum value varies from data set to data set and can be anything from 0 to 10 percent. As an alternative to RMS_{90}, the mathematical derivation is given of a more rigorous and reliable "RMS methodology" for measuring accuracy, known as the weighted RMS (RMS_W). It is shown [*2*], using real data, that RMS_W provides a reliable estimate for the 67 percent confidence radius. RMS_W has the further advantage that it takes into account the size of the predicted confidence region for each measurement. A grossly inaccurate measurement with a very small error prediction will be a major contributor to this overall statistics, whereas it may well be unjustifiably ignored when RMS_{90} is used.

It is therefore suggested that no measurements should be rejected in the computation of accuracy and that the RMS_W method be adopted as the basis for the FCC requirement.

Pursuant to Commission's Rule Section 1.419, one copy of this electronic filing is provided for inclusion in this docket.

If you, or anyone else, have further questions relating to this filing please do not hesitate to contact me at +44 1223 326900

Yours faithfully

CAMBRIDGE POSITIONING SYSTEMS LTD.

Andrew Pickford

Marketing Development Director

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References

[1] The home page of the T1P1.5 standards group may be found at: http://www.t1.org/t1p1/_p15home.htm.

[2] Cambridge Positioning Systems Limited, 1999, *The Use of a Weighted RMS Function to Reliably Represent the Distribution of Location Measurements*, Attachment 1.

[3] Public Notice DA 99-1049, *Wireless Telecommunications Bureau Requests Targeted Comment on Wireless E-911 Phase II Automatic Location Identification Requirements on CC Docket No. 94-102*, June 1, 1999.

[4] Cross, 1990, *Advanced Least Squares Applied to Position Fixing*, ISBN 0907382061.

[2] Cambridge Positioning Systems Limited, 1999, *Results from medium scale trial of an E-OTD system in an urban environment,* Attachment 2.

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Attachment 1 (11 pages):

The use of a weighted RMS function to reliably represent the distribution of location measurements

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1 Introduction

An E-OTD method of positioning has been developed which incorporates an enhanced least squares algorithm.

The results of the trial conducted by Cambridge Positioning Systems Ltd. (CPS) presented by Brice [1] consist of more than 9,000 measurements from 94 locations in Cambridge, UK. No measurements were ignored in the evaluation of trial results. 83.2 per cent were within 125 m of the actual position and 2 per cent were >250 m. RMS_{90} (defined in [4]) was 79 m, whereas 67 per cent of the measurements fell within 98 m.

Positional accuracy is most affected by multipath, and a poor geometrical disposition of the BTSs with respect to the handset. This paper addresses mathematically the effects that multipath and geometry have on the distribution of radial errors we observe. It explains why, when analyzing data taken from a plurality of sites, RMS on its own is not a good statistic for the 67 per cent confidence radius. RMS_{90} is better, but we propose a statistically based and more rigorous estimate.

2 The Trial System

CPS has deployed a city-wide commercial trial, based on its E-OTD technology, encompassing the entire urban and suburban areas of Cambridge, UK. The primary objective of the trial is to validate the commercial opportunities of LCS services in a realistic environment on a live network with a variety of users, including existing subscribers.

The dimensions of the commercial trial are:

1. 14 LMU sites;
2. 48 BTS cells (mix of sectored and omni-directional sites);
3. trial area = 150 km^2 centred on Cambridge city centre;
4. 250 MSs.

It should be noted that the ratio of LMU to BTS sites is not indicative of a full scale system. The small area of the trial system inherently has a large border area requiring extra LMUs.

The MS receives signals from, and measures timings on, a maximum of 7 BTSs. These are the serving cell and a maximum of six BTSs in its neighbour list, depending upon how many are visible. In some cases, the measurements relate to co-located transmitting antennas and are redundant in determining the MS's position. They nevertheless serve to average out errors.

3 Analysis of Results

3.1 Summary

In order to model the position fixes described by Brice in [1], we start with an elliptical Gaussian, characterized by its central coordinates, major and minor axes and an angle of rotation. It is shown, with reference to results [1], that in some cases this is indeed an adequate model, in which the RMS gives a good estimate for the 67 percent confidence region.

The key factor in this discussion is that, except in the special cases mentioned above, **a set of position fixes cannot be modeled as having been drawn from a single, simple distribution**. The shape of any realistic distribution has to depend on the geometrical disposition of the handset with respect to the BTSs used in the position calculation. The single distribution model can therefore only be used on position measurements taken from a fixed location (and sometimes not even then, if the BTS list is unstable).

Each position measurement is treated as having been drawn from its own unique elliptical distribution. The shape of this distribution can be predicted from the covariance matrix in x and y which falls naturally out of any least squares position calculation. Cross (1990) [3] gives a derivation of the least squares solution and discusses covariance matrices in detail.

The RMS has been shown to be an adequate statistic when analyzing data from a single elliptical distribution. Using this fact, a weighted RMS calculation is devised which scales the results so they are all drawn from the same distribution. This approach proves very successful in modeling our results.

3.2 Simplistic "circular" Model

Consider a set of N position fixes, $\{r_n\}$ with n = 1..N, where $r_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ is the two-dimensional vector position of the nth fix, all being taken at a single known location $\Re = \begin{pmatrix} X \\ Y \end{pmatrix}$. For simplicity we set $X = Y = 0$. If the effects of geometry and multipath described above did not exist, we might expect a Gaussian distribution of position fixes, centred on the actual position, given by

$$P(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(\frac{-\left(x^2+y^2\right)}{2\sigma^2}\right).$$

This "circular" distribution $P(x,y)$ is shown in figure 3.1.

It can easily be shown by changing to polar coordinates (r,θ) and by integrating over θ that the expected distribution of radial errors is given by

$$P(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right).$$

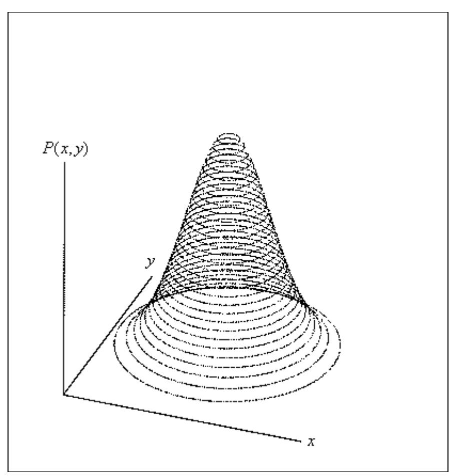

Figure 3.1

Not unexpectedly, **the distribution of radial errors is not Gaussian**. (See Figure 3.3 for a plot of $P(r)$).

The RMS of the radial errors is $\sqrt{\int_0^\infty r^2 P(r)dr} = \sqrt{2}\sigma$.

The confidence level given by RMS is

$$\int_0^{\sqrt{2}\sigma} P(r)dr = 1 - e^{-1} = 63.2 \text{ per cent.}$$

The 67 per cent region is, in fact, bounded by a circle of radius $(2\ln 3)^{1/2}\sigma = (\ln 3)^{1/2} \times \text{RMS}$. Hence, this simple model predicts that, **when analyzing data taken from a single location**, The E911 mandate is met with an RMS of 119 metres or better.

The results in Figure 3.2 (ref [1]) display a circular distribution of measurements centred on the actual position. It is therefore expected that the above model should apply.

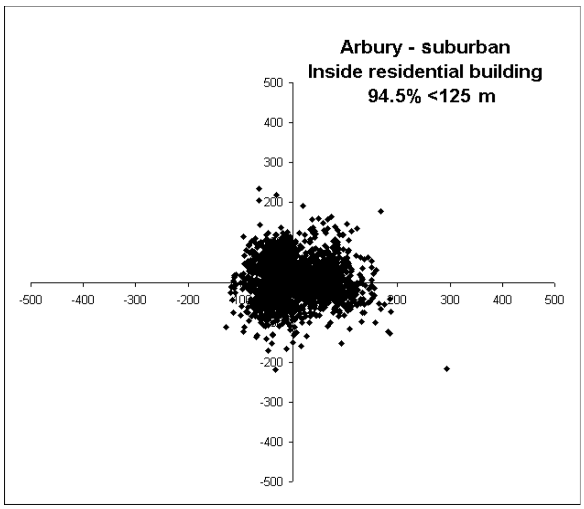

Figure 3.2

The RMS of the radial errors is measured as 73 metres. From this we predict:

	predicted	observed
Percentage of fixes within RMS:	63.2%	64.7%
Percentage of fixes within $\sqrt{\ln 3} \times$RMS:	66.7%	67.8%

Table 3.1

The radial errors should have the following distribution:

$$P(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right),$$

where σ = (RMS)/√2 = 52 m.

In figure 3.3, this predicted distribution is plotted against a histogram of the observed results. A close fit between the predicted and measured distribution is observed.

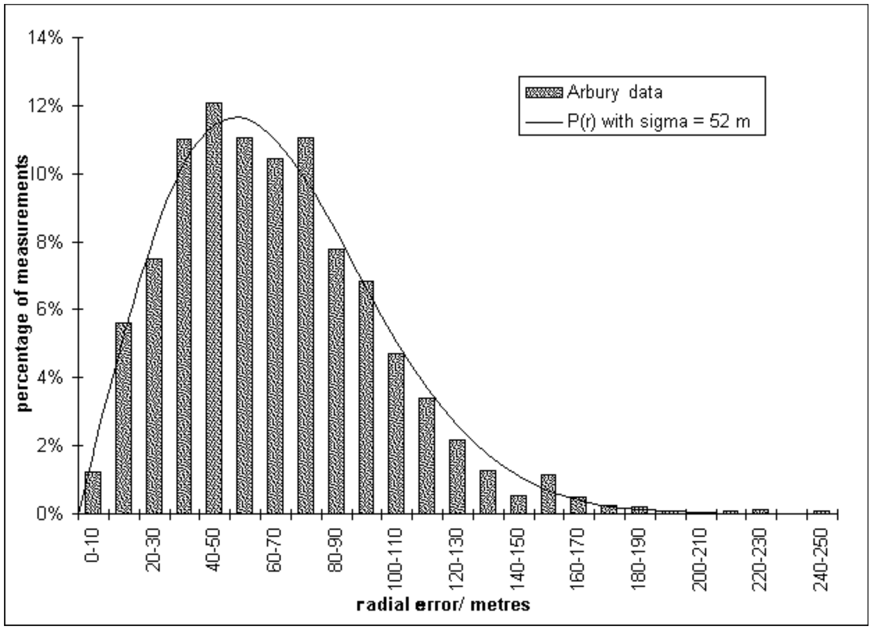

Figure 3.3. The observed and predicted, spread of measurements.

3.3 General "Elliptical" Model for Data from a Single Location

Many datasets were more severely affected by multipath and BTS geometry than the data plotted in Figure 3.2. In these cases a better model for the distribution of position measurements is an ellipse (not necessarily centred on the actual position).

To model this we continue from the definitions in section 3.2 and further define:

$$\delta r_n = r_n - \Re = \begin{pmatrix} \delta x_n \\ \delta y_n \end{pmatrix};$$

$$\mu_x = \sum_n \delta x_n / N \quad ; \qquad \mu_y = \sum_n \delta y_n / N;$$

$$\sigma_x = \sqrt{\sum_n (\delta x_n - \mu_x)^2 / N}\ ; \qquad \sigma_y = \sqrt{\sum_n (\delta y_n - \mu_y)^2 / N}\ ;$$

$$\sigma_{xy} = \sum_n (\delta x_n - \mu_x)(\delta y_n - \mu_y)/N.$$

If it is assumed that δr_n are actually samples from an elliptical Gaussian distribution, then the major and minor axes of that ellipse the eigenvalues of $\mathbf{C}_x^{-1}$,

where $\mathbf{C}_x$ is the covariance matrix of the data in *x* and *y* given by $\begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$.

It follows that

$$\sigma_{Major} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2}}{2}} \text{, and}$$

$$\sigma_{Minor} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 - \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2}}{2}} \text{.}$$

In terms of the cartesian coordinates, x and y, this elliptical probability distribution is given by

$$P(x,y) = \frac{1}{2\pi\sqrt{\sigma_x^2\sigma_y^2 - \sigma_{xy}^2}} \exp\left(\frac{-\left(\sigma_y^2(x-\mu_x)^2 + \sigma_x^2(y-\mu_y)^2 - 2\sigma_{xy}(x-\mu_x)(y-\mu_y)\right)}{2(\sigma_x^2\sigma_y^2 - \sigma_{xy}^2)} \right)$$

The radial distribution of errors is

$$P(r) = \int_0^{2\pi} r.P(r.\cos\theta, r.\sin\theta)d\theta \text{,}$$

but this distribution cannot be evaluated symbolically in general. However, the predicted RMS of the radial errors for this distribution can be calculated as

$$\text{RMS} = \sqrt{\int_0^{\infty} r^2 P(r)dr} = \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x^2+y^2)P(x,y)dxdy} = \sqrt{\sigma_x^2 + \sigma_y^2 + \mu_x^2 + \mu_y^2} \text{.}$$

Numerical evaluations of $P(r)$ can be made in individual cases. The data plotted in Figure 3.4 is modeled in this way below.

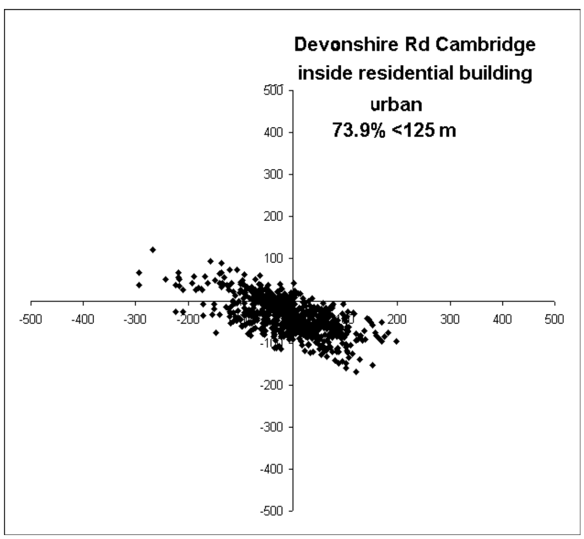

Figure 3.4

Using the equations defined above, the following statistics have been evaluated for the Devonshire Road data.

	Value
μ_x	-10 metres
μ_y	-36 metres
σ_x	73 metres
σ_y	42 metres
σ_{xy}	-1982 metres^2

Table 3.2

$P(x, y)$ for these parameters is plotted in Figure 3.5 below, in which the circle marked is of radius 125 m and centred on the actual position.

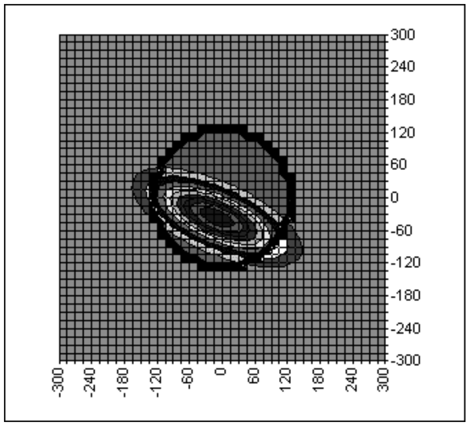

Figure 3.5 A numerical approximation to the distribution of Figure 3.4

$P(r)$ has been evaluated numerically and plotted in figure 3.6

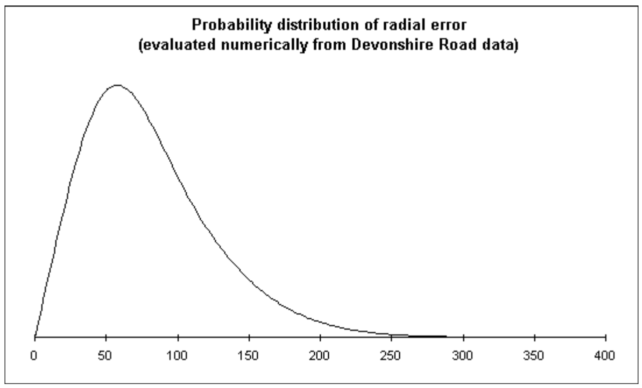

Figure 3.6 A numerical approximation of the distribution of radial errors in Figure 3.4.

It is used to predict the percentage of expected results within 125 metres and also the 67 per cent confidence level. The predicted and observed statistics are shown in Table 3.3

	predicted	observed
results < 125 m	85%	86%
67% of results are within [metres]	92	89

Table 3.3

This analysis was performed on all the other “single location” results in [1]:

	RMS [metres]	predicted results < 125 [metres]	predicted 67% radius [metres]	observed results < 125 [metres]	observed 67% radius [metres]
Devonshire Rd	85	85%	92	86%	89
Arbury	74	94%	76	95%	76
Covent Garden	67	94%	66	94%	68
Cavendish	145	45%	154	57%	137

Table 3.4

It is clear from these results, that this model works well for some of the positions but badly for the Cavendish data. In this case, the elliptical distribution predicted by $P(x, y)$ is far larger than that observed. The model used is therefore incorrect for these measurements, i.e.

even for data taken from a single location, the results cannot always be assumed to have been drawn from a single Gaussian distribution.

In the cases where the model does fit, however, RMS is a good approximation to the 67 per cent confidence radius.

3.4 The Weighted RMS Solution

The key to understanding and analyzing this data comes from the predicted confidence ellipse (described by Read (1999) [2]). It is shown by Cross [3] how the covariance matrix (used in the least squares approach to calculating positions) can take into account the geometry of the surrounding BTSs and predict such an ellipse.

The Cavendish data has been observed in section 3.3 not to fit the "single distribution" model. On closer analysis we note that 99.3 per cent of the measurements had ellipses with radii less than 500 metres while the remaining 0.7 per cent had ellipses with major axes larger than 1800 metres. These latter 9 measurements are marked with crosses in Figure 3.7. It is clear that we have two distinct distributions, explaining why the single distribution model failed. (In the 0.7 per cent subset, a vital BTS was not seen by the handset giving very poor geometry.)

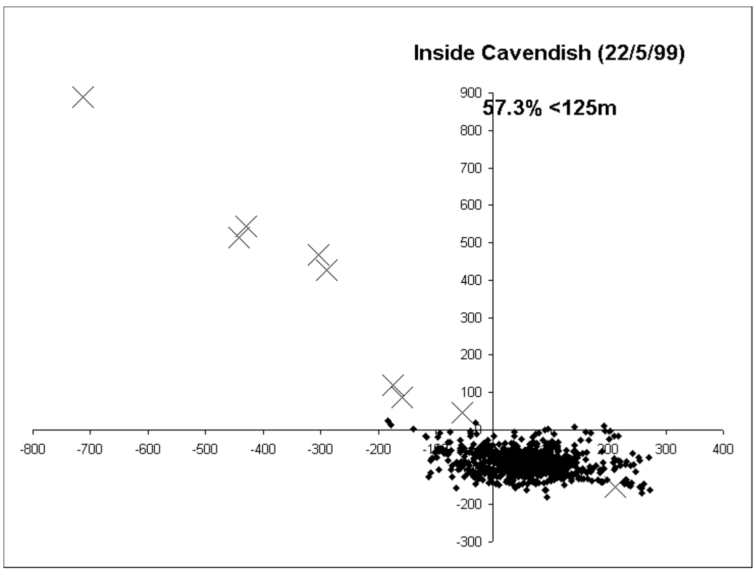

Figure 3.7

One approach (which forms the basis for the RMS_{90} calculation) is to ignore any data which doesn't fit the distribution. In fact this gives very good results for the above data[1] but it is not a general solution. When analyzing data from multiple locations, and hence multiple distributions of position fixes, it is clearly impossible, or at least statistically ill-advised, to ignore the data which doesn't fit the model.

We are able however to make use of the observation made in section 3.3: **"In the cases where the elliptical model does fit, the RMS is a good approximation to the 67 per cent confidence radius"**.

We consider every point on the above plots to have being drawn from its own unique elliptical distribution, the defining parameters of which are assumed to equal those of the predicted confidence ellipse for that measurement. It is therefore possible to scale, or weight,

[1] This predicts 54 per cent of results being within 125 m, compared with 57 per cent observed.

each point so the combined set of measurements appear to be drawn from a single distribution. Once this is done, the RMS approximation should be valid. Therefore, for a set of *N* measurements, 67 per cent of them should fall within RMS_w given by

$$RMS_w = \sqrt{\frac{\sum_{n=1}^{N} r_n^2 / w_n^2}{\sum_{n=1}^{N} 1 / w_n^2}}\ .$$

A set of position fixes, drawn from an elliptical distribution (assumed to be centred on the actual position), with orthogonal widths of σ_x and σ_y, have an RMS = $\sqrt{\sigma_x^2 + \sigma_y^2}$. (see section 3.3). For a single position fix from that distribution, an estimator for σ_x and σ_y is given by the major and minor axes of the predicted confidence ellipse. The weight, w_n, of the nth measurement is therefore chosen to equal $\sqrt{\sigma_{Major}^2 + \sigma_{Minor}^2}$, where σ_{Major} and σ_{Minor} equal the predicted major and minor axes of the 67 per cent confidence ellipse for that measurement.

RMS_w and RMS_{90} are compared in Table 3.5 with the observed 67 per cent confidence level (within which 67 per cent of all measurements lay).

	RMS_{90} [metres]	RMS_w [metres]	67% of tests fell within [metres]
Outdoor urban.	93	115	103
Covent Garden – indoor urban	56	67	68
Devonshire Road – indoor urban	75	93	89
Arbury – indoor suburban	62	72	76
Cavendish – indoor suburban	117	137	137
Overall Results	79	101	98

Table 3.5

4 Conclusion

The overall results from extensive field trials indicate (see Table 3.5) that the weighted RMS provides a reliable statistic and predictor of measurement distribution. Its statistical basis is more rigorous than that for RMS_{90} and, most importantly, **all measurement data is included in the calculation.**

It is proposed that RMS_w be adopted as a more realistic measure of the 67 percent confidence region.

5 References

[1] T1P1.5/99-390, Results from medium scale trial of an E-OTD system in an urban environment, CPS, 1999.
[2] T1P1.5/99-158, Advanced Positioning Algorithms and Elliptical Confidence Regions, CPS 1999.
[3] Advanced least squares applied to position-fixing, ISSN 0260-9142, P A Cross 1990
[4] T1P1 5/99-185r0 CR against GSM 05.05 version 6.3.0 (Motorola, Inc)

Attachment 2 (10 pages):

Results from medium scale trial of an E-OTD system in an urban environment

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1 Introduction

Cambridge Positioning Systems Ltd. (CPS) use an E-OTD method of positioning, incorporating an enhanced least squares algorithm.

A wide area trial network that includes a variety of operational environments provides a highly realistic basis for evaluating the commercial basis on which location-based services could be provided. A trial system also allows system performance, including location accuracy to be determined. Assessments of measured location indicated that that RMS_{90} was a poor measure of the 67 percent confidence region and that a more rigorous method was required.

The scope of the tests performed on a trial network consisted of more than 9,000 measurements taken at 94 locations in Cambridge, UK. No measurements were ignored in the evaluation of the test results presented below.

2 Trial System

CPS Ltd has deployed a city-wide commercial trial, based on its E-OTD technology, encompassing the entire urban and suburban areas of Cambridge, UK. The primary objective of the trial is to validate the commercial opportunities of LCS services in a realistic environment on a live network with a variety of users, including existing subscribers.

The dimensions of the commercial trial are:

1. 14 LMU sites;
2. Approximately 48 BTS cells (mix of sectored and omni-directional sites);
3. Trial area = 150 km^2 centred on Cambridge city centre, and
4. 250 MSs

It should be noted that the ratio of LMUs to BTS sites is not indicative of a full scale system. The small area of the trial system inherently has a large border area requiring extra LMUs.

The MS receives signals from, and measures timings on, a maximum of 7 BTSs. These are the serving cell and a maximum of six BTSs in its neighbour list, depending upon how many are visible. In some cases, the measurements relate to co-located transmitting antennas and are redundant in determining the MS's position. They nevertheless serve to average out errors.

3 Results

A grid with a 500 x 500 metre pitch was overlaid on the greater Cambridge area to encompass all possible urban and suburban trial environments within a 150 km^2 'area. Measurement sets from street-level, stationary MSs were gathered from locations positioned nearest to the centre of each grid-cell. Actual positions were accurately determined with an accuracy within 2 metres using local Ordinance Survey mapping data.

Test results from the environments: urban (A and B), suburban, indoor urban (A and B) and indoor suburban are presented in sections 3.1, 3.2 and 3.3. A summary is given in section 3.4.

3.1 Urban

The following results were taken from several outdoor, street-level locations shown in Figs. 1 and 2 below.

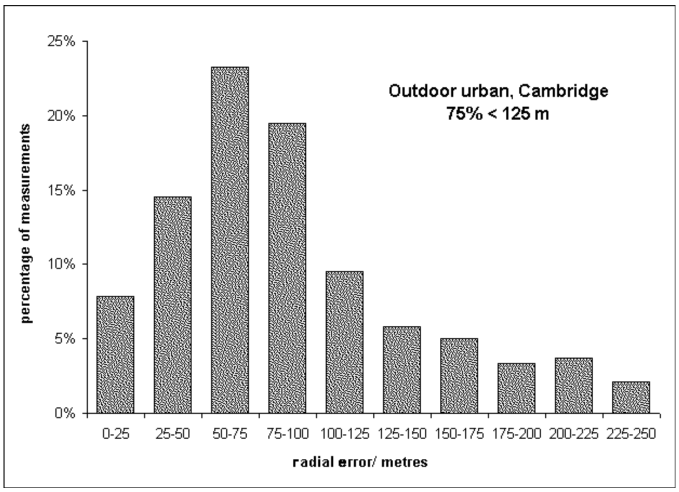

Figure 1

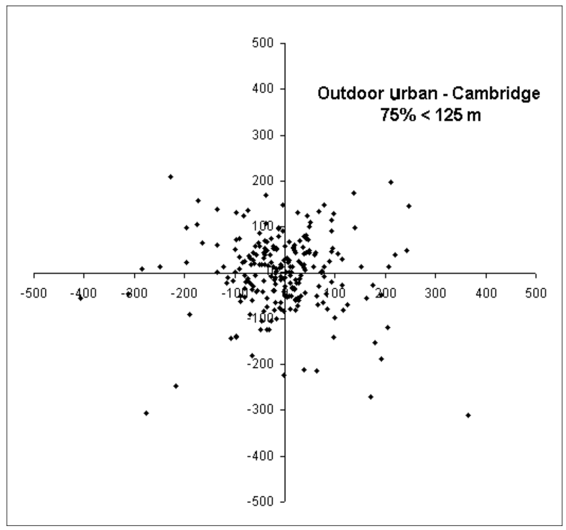

Figure 2

3.2 Indoor Urban

Over 2000 measurements were made from several indoor urban sites in the centre of Cambridge. Two locations presented separately below represent typical results (including worst case location measurements).

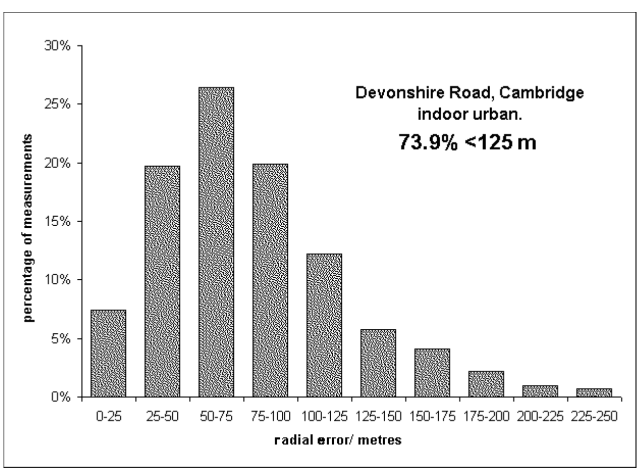

Figure 3

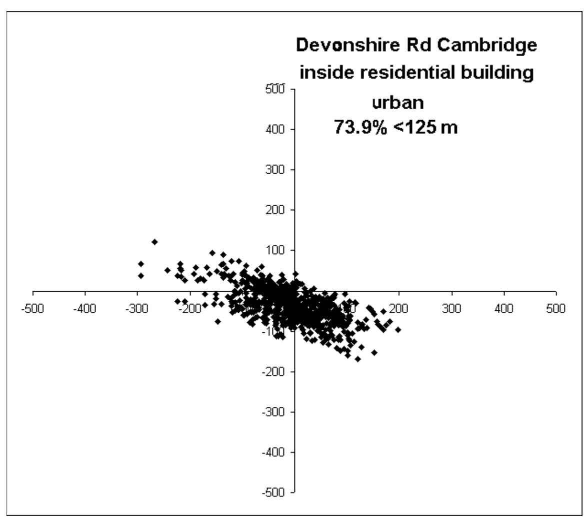

Figure 4

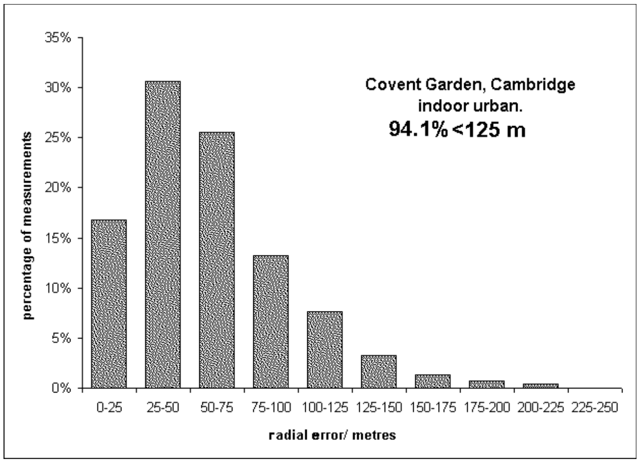

Figure 5

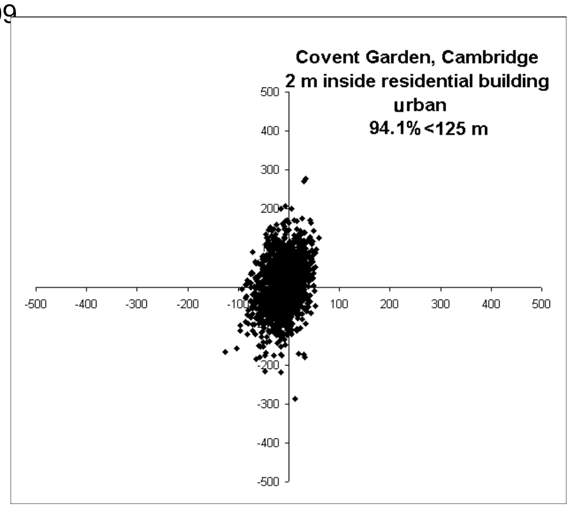

Figure 6

3.3 Indoor suburban

Over 2000 measurements were made from several indoor suburban sites in the centre of Cambridge. Two locations, presented separately below, represent typical results (including worst case location measurements).

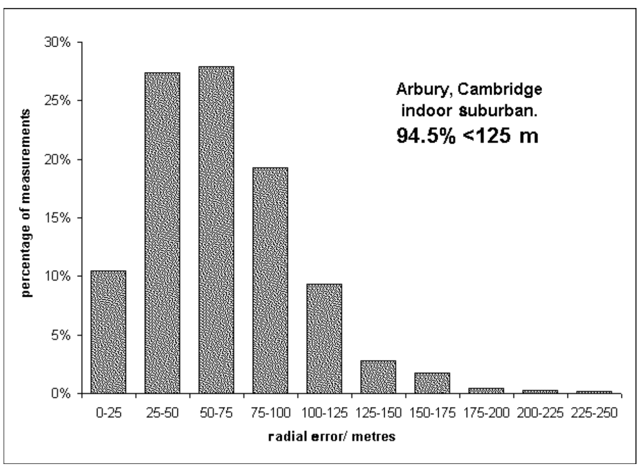

Figure 7

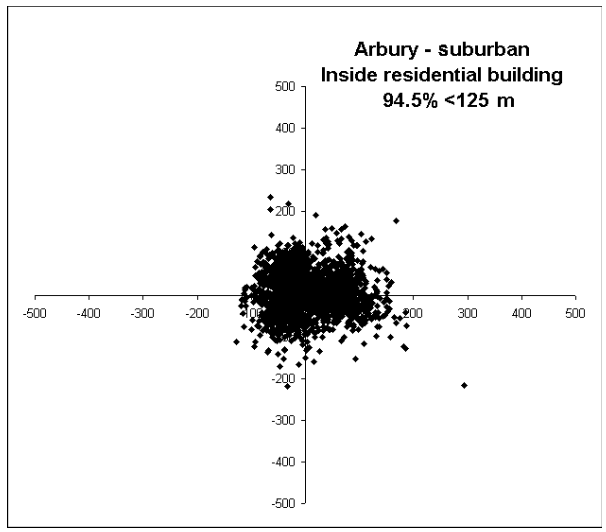

Figure 8

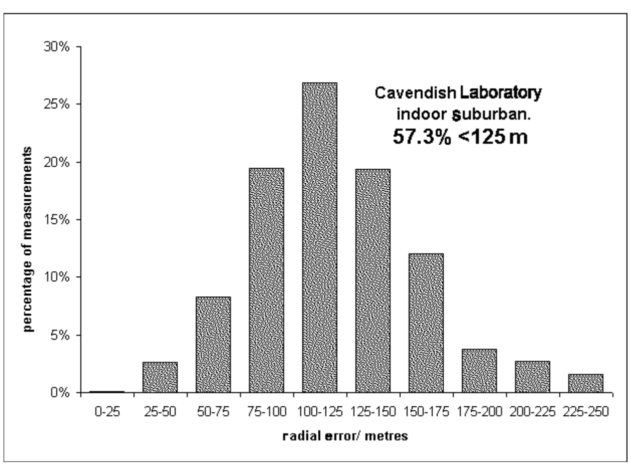

Figure 9

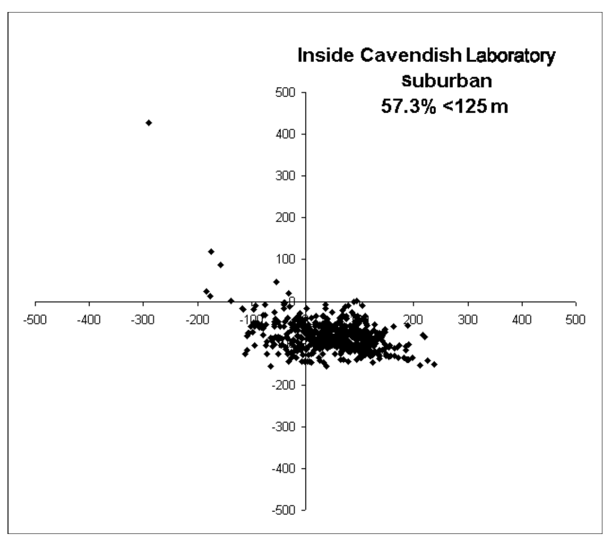

Figure 10

Note that Cavendish Laboratory is a concrete structure. The MS was located more than 6m from the nearest peripheral wall.

3.4 Overall Results

Table 3.1 below summarises the results presented in sections 3.1, 3,2 and 3.3 above.
Two methods of determining RMS were adopted: RMS_{90} (as defined in [3]) and RMS_w[2] and listed alongside the 67 per cent threshold.

	RMS_{90} [metres]	RMS_w [metres]	67% of measurements within [metres]
Urban.	93	116	103
Covent Garden – indoor urban	56	67	68
Devonshire Road – indoor urban	75	92	89
Arbury – indoor suburban	62	72	76
Cavendish Laboratory – indoor suburban	117	137	137
All Results	79	101	98

Table 2.1

An alternative presentation of the results considers the percentage of measurements within 125 metres as shown in Table 3.2 below.

	Measurements within 125 metres
Overall Results	83 %
Outdoor urban.	75 %
Indoor urban	85 %
Indoor suburban	86 %

Table 2.2

[2] The weighted RMS (RMS_w) has been shown by Brice [1] to be a far more reliable and mathematically rigorous statistic than RMS_{90}. For N measurements, $RMS_w = \sqrt{\dfrac{\sum_{n=1}^{N} r_n^2 / w_n^2}{\sum_{n=1}^{N} 1 / w_n^2}}$, where w_n = the average of the major and minor axes of the 67 per cent confidence ellipse <u>predicted</u> for the nth measurement, and r_n is the observed radial error on that measurement.

3.5 Ellipses of Confidence

A 67 per cent ellipse of confidence was calculated for every location measurement as described by Read (1999)[2]. The chart below demonstrates the expected dependency (highlighted with a line having a intercept through the origin), between the major axis and the true radial error.

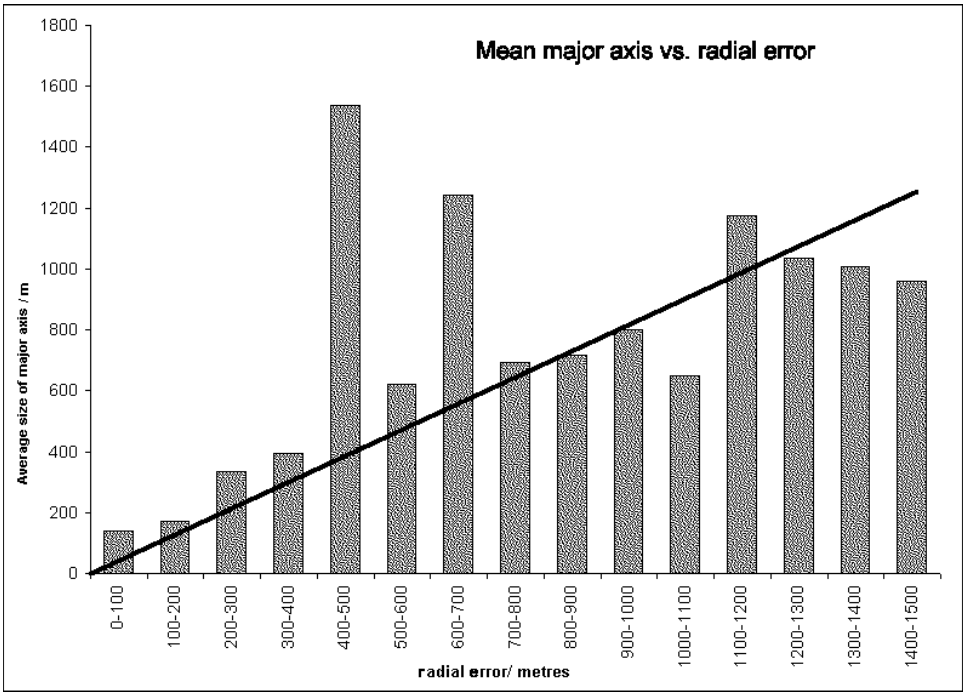

Figure 11

4 Summary and Conclusions

4.1 Aggregated Results

The results of CPS's trials on its commercial E-OTD trial network, shown below, consist of more than 9,000 measurements from 94 locations in Cambridge. Environments ranged from indoor urban to outdoor suburban.

Aggregated results indicated that 83.2 per cent were within 125 m of the actual position and 2 per cent were >250 m. RMS_{90} was 79 m but 67 per cent of the measurements fell within 98 m. The weighted RMS (RMS_w) was 101m.

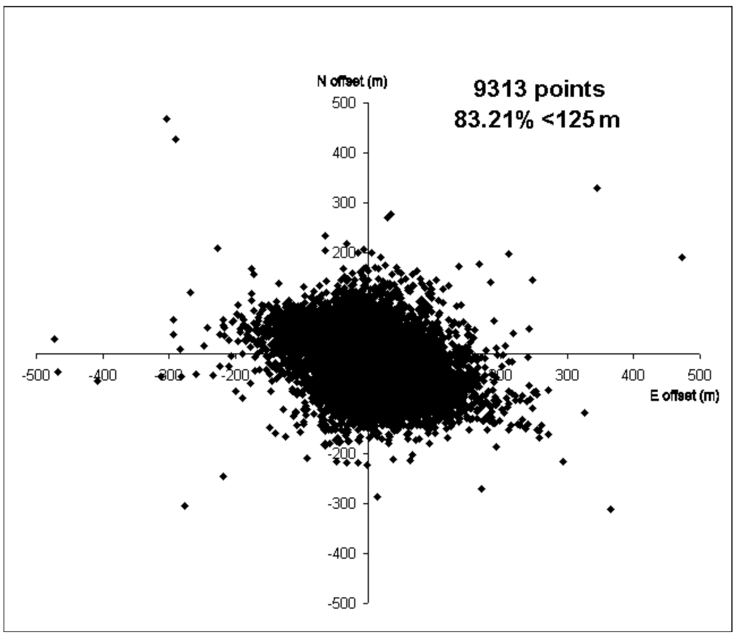

Figure 12

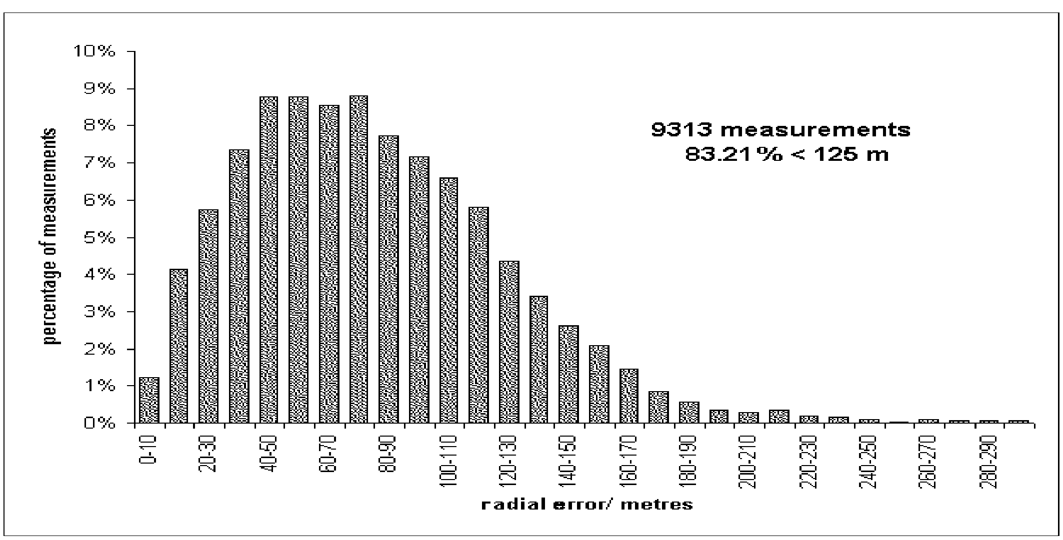

Figure 13

4.2 RMS_{90} or RMS_w

In almost all cases, the weighted RMS calculation (the basis for which is described in detail by Brice, 1999 [1]) provides a reliable statistic. Furthermore the results show that the statistical basis for RMS_w is more rigorous than that for RMS_{90} and, most importantly, **without eliminating any measurement data from the calculation.**

4.3 Conclusions

Aggregated results on the commercial E-OTD trial network indicated that 83.2 per cent were within 125 m of the actual position and 2 per cent were >250 m. RMS_{90} was 79 m but 67 per cent of the measurements fell within 98 m. The weighted RMS (RMS_w) was 101m.

5 References

[1] T1P1.5/99-389, The use of a weighted RMS function to reliably represent the distribution of location measurements, CPS, 1999

[2] T1P1.5/99-158, Advanced Positioning Algorithms and Elliptical Confidence Regions, CPS, 1999.

[3] T1P1 5/99-185r0 CR against GSM 05.05 version 6.3.0 (Motorola, Inc)